## Issue partial fractioning with GiNaC

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## 1 Introduction

Let us investigate here, when the function of GiNaC, sqrfree parfrac gives wrong results.
I first noticed this error, for terms of the form

$$
\begin{equation*}
\frac{1}{a} \frac{x^{b}}{(x-1)^{c}} . \tag{1}
\end{equation*}
$$

Applying this to some particular example, using partial fractioning this should give

$$
\begin{equation*}
\frac{1}{4} \frac{x^{4}}{x-1}=\frac{1}{4}\left(x^{3}+x^{2}+x+\frac{1}{x-1}+1\right) . \tag{2}
\end{equation*}
$$

The result outputed by GiNaC was

$$
\begin{equation*}
\frac{1}{4}\left(x^{3}+x^{2}+x+1\right)+\frac{1}{x-1} . \tag{3}
\end{equation*}
$$

As you can see, the prefactor missing in front of the $\frac{1}{x-1}$ term is missing. That this is not simply an output error can be seen from plugging a value for the $x$ term, here I used $x=4$.

I investigated this and the following terms all give wrong result.

$$
\begin{align*}
& \frac{1}{4} \frac{x^{4}}{(x-1)^{2}},  \tag{4}\\
& \frac{1}{4} \frac{x^{3}}{(x-1)^{3}},  \tag{5}\\
& \frac{1}{3} \frac{x^{3}}{(x-1)^{2}},  \tag{6}\\
& \frac{1}{5} \frac{x^{2}}{(x-1)^{3}} . \tag{7}
\end{align*}
$$

Notice however, that terms of the form (no denominator present for the numerical prefactor) give a correct result:

$$
\begin{equation*}
3 \frac{x^{3}}{(x-1)^{2}} . \tag{8}
\end{equation*}
$$

As you can see, there is an emerging pattern. We always see that the term $1 /(\ldots)$ initially present ín the expression is missing the (numerical) prefactor, if the prefactor is a ratio.

The code I used was

```
#include <iostream>
#include <complex>
#include <math.h>
#include <ginac/ginac.h>
using std::cout;
using std::endl;
int main()
{
    GiNaC::symbol x("x");
    GiNaC::ex ex1 = pow (x,4)/(x-1)/4;
    GiNaC::ex ex2 = GiNaC::sqrfree_parfrac(ex1,x);
    cout << "the input is ex1 = " << ex1 << endl;
    cout << "the result after partial fractioning: "}<<<<ex2 << endl
    cout << "this is what you get after x=4, in ex1 = "<< ex1.subs(x==4)<< endl;
    cout << "insert number:, say x = 4, which gives wrong result "
    << ex2.subs(x==4)}<< endl
}
```

And I modified the term ex1 accordingly.

